Discuss one of the following subjects at your choice, giving a detailed treatment of the concrete examples involved.

1) A rigid body with a symmetry axix (*e.g.* a homogeneous cube) moving in space is a completely integrable system with a large symmetry group.

In this connection, discuss the role of symmetry and complete integrability in Mechanics.

2) Discuss the classification of second order linear partial differential equations by using the classical examples of Mathematical Physics.

3) Provide expression for the mean energy $\bar{E}$, the mean sqaure energy $\bar{E}^2$ and the mean square energy fluctuation $\bar{\Delta} E^2$, $(\Delta E = E - \bar{E})$ for a system (microscopic or macroscopic) in contact with a heat bath at a temperature T. Apply the results to a harmonic oscillator and to a collection of N identical harmonic oscilllators. Discuss the results.

4) Consider the operator:

$$H = -\frac{1}{2}\frac{d^2}{dx^2} + \frac{1}{2}x^2 + \gamma \sin |x|$$

defined on twice–differentiable functions on $\mathbf{R}$ with compact support. γ is a real parameter satisfying $|\gamma| < 1/2$.

The operator H is closable as an operator in $L^2(\mathbf{R}, dx)$and its closure $\bar{H}$ is self–adjoint.

Prove that $\bar{H}$ has only point spectrum and establish the asymptotic behaviour of the eigenvalues λ_n for large n as well as the dimension of the eigenspace corresponding to the first n eigenvalues. Prove moreover that for β sufficiently large, the operator $(\bar{H} + \beta I)^{-1}$ is of Hilbert–Schmidt class.

Which are the other values of the coplex parameter z for which the operator $(\bar{H} + zI)^{-1}$ is Hilbert–Schmidt?

Using the symmetry of the function $\sin |x|$ is it possible to extend the previous results to the case $\gamma < 1$?.

Optional

Prove the above statement about the closability of H and the self–adjointness of $\bar{H}$.