

Introduction to Linear Differential Equations in the Complex Domain – 10 lectures, 20 hours

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The aim of the course is to provide basic notions on linear systems of differential equations in the complex domain and monodromy data, which play an important role in modern mathematical physics, for example in the field of integrable systems.

- Review of existence and uniqueness theorems in the complex domain.
- Linear systems
- Singularities and monodromy
- Classification of isolated singularities of linear systems (first and second kind).
- Linear systems with singularities of first kind (Fuchsian systems).
- Linear equations of order n . Riemann and Gauss equations.
- Review of Poincaré asymptotics.
- Linear Systems with singularities of the second kind.
- Stokes phenomenon (some examples, such as Bessel equation and Airy equation).
- Global description. Monodromy data.
- If time allows (?), applications to non linear equations will be sketched (such as the method of isomonodromy deformations).

Prerequisites: Complex analysis, theory of analytic functions in one complex variable (see Reference 4. below).

Basic References

1. W. Wasow: *Asymptotic Expansions for Ordinary Differential Equations*.
2. E.A. Coddington, N. Levinson: *Theory of Ordinary Differential Equations*.
3. E.L. Ince: *Ordinary Differential Equations*

4. V.I. Smirnov: *A course of higher mathematics. Vol. 3. Part 2: complex variables, special functions*
5. K. Iwasaki, H. Kimura, S. Shimomura, M. Yoshida: *From Gauss to Painlevé.*
6. W. Balser, W.B. Jurkat, D.A. Lutz: "*Birkhoff Invariants and Stokes' Multipliers for Meromorphic Linear Differential Equations*". *Journal of Mathematical Analysis and Applications* **71**, 48-94 (1979).
7. A. Fokas, A.R. Its, A.A. Kapaev, V.Y. Novokshenov: *Painlevé Transcendents, The Riemann-Hilbert approach.*
8. Y. Sibuya: *Linear Differential Equations in Complex Domain; Problems of Analytic Continuation.* Translations of Mathematical Monographs 82, AMS.