SISSA - Università di Trieste
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Problem 1

Find a family $u(x,t;s)$, where $s \geq 0$ is a parameter, of solutions of the Cauchy problem

$$\begin{aligned} u_{tt}(x,t;s) &= u_{xx}(x,t;s), \quad -\infty \leq x \leq \infty, \quad t > 0, \\ u(x,t;s)|_{t=s} &= 0, \\ u_t(x,t;s)|_{t=s} &= \cos(x). \end{aligned}$$

Problem 2

Find solution $u(y,z)$ of the inhomogeneous Cauchy problem

$$\begin{aligned} 4u_{zz}(y,z) + 4u_{yz}(y,z) &= \cos(y - \frac{z}{2}), \quad -\infty \leq y \leq \infty, \quad z > 0, \\ u(y,z)|_{z=0} &= 0, \\ u_z(y,z)|_{z=0} &= 0. \end{aligned}$$

Hint: Reduce to the canonical form and use the problem 1.

Problem 3

Find solution $u(x,t)$ to the Cauchy problem

$$\begin{aligned} u_t(x,t) &= u_{xx}(x,t), \quad x \in (-\infty, \infty), \quad t > 0, \\ u(x,t)|_{t=0} &= e^{-\frac{1}{2}x^2}. \end{aligned}$$