C* algebras and their many K-theory groups

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BRUTAL SYNOPSIS

OF THE BIG PICTURE

- Complex C*-algebras
  - abstract view: axioms
  - concrete view: $\subset B(H_C)$
- Real C*-algebras
  - abstract view: axioms
  - concrete view: $\subset B(H_R)$
  - antiautomorphism view
- Foundations of K-theory: Bott periodicity, or the homotopy groups of $GL_n(A)$.
- complex K-theory
  - homotopies of projections,
  - homotopies of unitaries
- real K-theory
  - homotopies of projections,
  - homotopies of unitaries with extra conditions — 7 different flavours!
- algebraic K-theory

- KK-theory
  - homotopy classes of 'absorbing' Busby maps (absorbing extensions)
  - homotopy classes of Fredholm triples
    - bounded
    - unbounded
- Kasparov products
  - Kasparov's original definition
  - Connes-Skandalis bounded connection definition
  - unbounded version of the connection definition
  - as connecting maps in K-theory
- relationship between real and complex case
  - very simple?
  - or not?
  - products with "small" elements
- The UCT: a refinement of the connecting maps picture