NCG (NONCOMMUTATIVE GEOMETRY), II

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In the part I of the course, we have seen some decent examples of noncommutative spaces in NCG following the spectral triple paradigm. On the other hand, many commutative spaces, such as foliations, space with symmetries (i.e. with group actions) are also main sources of noncommutative spaces. The question of upgrading the corresponding index related questions inspired the development of many basic tools in NCG, such as KK-theory, cyclic (co)homology and the notion of spectral triples. It has turned out to be a powerful approach with numerous applications in geometry, topology and representation theory.

The course is devoted to an introduction to the celebrated Atiyah-Singer (A-S) local index theorem via the heat kernel method. The two books [BGV92] and [Roe99] are popular references on the subject. We plan to cover [BGV92, Ch.1-4] and the related chapters in [Roe99] are 1-8 and 11-13. Here is the outline, beginning with background materials in differential geometry:

- Exterior algebras and Clifford algebras;
- Spin groups and spin bundles;
- Connection theory on principle bundles which is needed
  - to construct characteristic classes (the Chern-Weil method);
  - to lift Levi-Civita connections to spinorial connections.

More advanced topics (highlights in [BGV92, Ch.1-4]) include:

- Gauss-Bonnet-Chern theorem, Hirzebruch signature theorem and Hirzebruch-Riemann-Roch theorem as applications of the general A-S local index theorem for Dirac operators;
- The asymptotic expansion of the heat kernel and Weyl's asymptotic formula for the eigenvalues (a.k.a Weyl's law);
- Getzler symbol for differential operators and heat kernels (a.k.a Getzler's rescaling trick) and the proof of the A-S local index theorem;
- Mehler's formula for harmonic operators which is required in the last step of the Getzler's proof of the local index theorem.

The course shall be self-contained only assuming basic knowledge in differential geometry and differential topology. Lecture notes will be distributed as the course moving forward. As a 20-hour course, it will take five weeks starting in the week of Jan 25, 2021. Classes are scheduled on Tuesday 11-1pm and Thursday 2-4pm.

References

[BGV92] Nicole Berline, Ezra Getzler, and Michele Vergne. *Heat kernels and Dirac operators*. Springer, 1992.
[Roe99] John Roe. *Elliptic operators, topology, and asymptotic methods*. CRC Press, 1999.

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