

NCG (Noncommutative Geometry), II

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In the second half of the course, we would like to illustrate some basic ideas and tools in NCG through one example: noncommutative two tori, the simplest but most revealing one. We shall explain in great detail how to adapt questions in differential geometry (metric structure and index problem) into an operator theoretical framework. The course is divided into three parts.

- Various constructions of noncommutative two tori:
 - Weyl quantization and θ -deformation.
 - Noncommutative quotients, that is, the “space of leaves” of Kronecker foliations. The goal is to introduce the Heisenberg modules and discuss Morita equivalence between noncommutative two tori.
- Riemannian aspects of NCG:
 - Connes’s spectral triple paradigm:
 - Notions of metric, complex structures, conformal structures, $\text{Spin}_{\mathbb{C}}$ -structures in the operator theoretical framework;
 - Twisted spectral triples, conformal change of metrics in [LM16, CM14];
 - Modular Gaussian curvature and Morita equivalence [LM16, CM14];
- Differential topological aspects of NCG:
 - Connes’s computation of Hochschild and cyclic cohomology on $C^{\infty}(\mathbb{T}_{\theta}^2)$, [Con85, Part II, §6];
 - Elliptic differential operators and index computation through pairings between K -theory and cyclic cohomology. The goal is to understand Connes’s computation in [Con80] and [Con94, Chapter 6];

We will mainly follow [Kha13], a quite friendly introduction, to explore deep topics like groupoids algebra, Morita equivalence, Hochschild and cyclic cohomology, Connes-Chern characters.

References

- [CM14] Alain Connes and Henri Moscovici. Modular curvature for noncommutative two-tori. *J. Amer. Math. Soc.*, 27(3):639–684, 2014.
- [Con80] A. Connes. C^* -algebres et géométrie différentielle. *CR Acad. Sci. Paris Sér. AB*, 290(13):A599–A604, 1980.
- [Con85] A. Connes. Non-commutative differential geometry. *Publications Mathématiques de l’IHES*, 62(1):41–144, 1985.
- [Con94] Alain Connes. *Noncommutative geometry*. Academic Press, Inc., San Diego, CA, 1994.
- [Kha13] Masoud Khalkhali. *Basic noncommutative geometry*. EMS Series of Lectures in Mathematics. European Mathematical Society (EMS), Zürich, second edition, 2013.
- [LM16] Matthias Lesch and Henri Moscovici. Modular Curvature and Morita Equivalence. *Geom. Funct. Anal.*, 26(3):818–873, 2016.