

# Noncommutative Geometry I

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2019

## Timetable and room

Every Monday 11:00 – 13:00 and every Friday 14:00 – 16:00 starting from Monday 14 October 2019 to (approximately) Monday 9 December. **Room** Boris Dubrovin Lecture room: 136.

## Course contents

### First part: $C^*$ -algebras

- Basic functional analysis, Banach spaces, linear operators
- Banach algebras, spectrum, Gel'fand transform, functional calculus
- $C^*$ -algebras and their properties
- Gel'fand duality between  $C^*$ -algebras and locally compact Hausdorff spaces
- Continuous functional calculus, positive elements, approximate units
- states and representations
- Gel'fand-Naimark-Segal (GNS) construction

### Second part: basic index theory

- vector bundles, modules and Serre-Swan correspondence, some informations on  $K$ -theory.
- Clifford algebras and Clifford modules
- Spin structures
- Construction of Dirac operators
- Basics of elliptic operators, elliptic regularity and index
- the Atiyah–Singer index formula

## Examination

The exam will consist of a seminar presentation on an advanced topic related to the course.

## Recommended literature

1. B. Blackadar, Operator algebras: Theory of  $C^*$ -algebras and von Neumann algebras, Encyclopaedia of Mathematical Sciences, vol. 13, Springer, 2006.
2. J. Conway, A course in functional analysis Second edition. Graduate Texts in Mathematics, 96. Springer-Verlag, New York, 1990.
3. J. Roe, Elliptic operators, topology and asymptotic methods (II edition). *Pitman research notes in Mathematics*, 395 1999.
4. H. B. Lawson and M-L. Michelsohn, Spin geometry. Princeton Mathematical Series, 38. Princeton University Press, Princeton